

Development of "Loco Boiler" - A Predictive Calculation for Boiler Design Analysis

introduction

This report describes the development of a numerical method for determining the performance of miniature coal fired, fire tube boilers starting with the boiler geometry and firing rate.

This has been done before, notably by Professor Bill Hall and some of my work follows his methods. I have chosen to build the model in the Excel spreadsheet format, which places considerable constraints on the mathematical methods used, but does make it accessible to almost everyone with a personal computer. I have also avoided the use of macros and "goal seek" in the solution of the relevant equations for ease of access and simplicity of use.

Optimising the boiler model without considering the remainder of the locomotive will not work for the following reasons:

- ❓ Efficient heat exchange in the tube bank requires pressure loss of the flue gases.
- ❓ Flue gas pressure loss is overcome by the blast pipe / chimney jet pump. More pressure loss in the tube bank ultimately requires a fiercer blast which is achieved by increased back pressure on the engine cylinders and hence loss of indicated power. Therefore, the performance of the boiler / cylinder / blast pipe / chimney combination is what matters.
- ❓ Similarly, provision of superheating reduces the evaporative capacity and efficiency of the boiler, but this is offset by improvements in the indicated cylinder power per unit mass of superheated steam.
- ❓ In order to optimise the balance between heat exchange efficiency and indicated power from the cylinders, the boiler and cylinder performance must be modelled.

At the outset of the project, it was thought that flue gas pressure loss, and hence draughting design would be a key issue. At the time of writing, the cylinder / blast pipe elements have not been numerically modelled. However, during the development, it has become clear that superheater design is more important for efficient design in miniature engines. It is the author's opinion that most model engines have insufficient superheat.

APPLICABILITY

The spreadsheet program "LOCO BOILER" can be applied to the analysis of coal fired, firetube type boilers with or without superheaters and radiant superheaters. Fireboxes of the conventional box style or marine style can be calculated. The program assumes the superheater is after the regulator in the steam circuit, and that superheater flues are in parallel with the firetubes in the flue gas circuit. The program is suitable for either vertical or horizontal firetube boilers.

The program will calculate firetube and superheater flue flows in both laminar and turbulent regimes as appropriate. The program can accommodate multiple superheater elements within a single flue, but not the 4 pass "Schmidt" style superheater element.

The program is written in a modular way and can be adapted to other boiler configurations. The author has adapted the program to consider "Sentinel" style boilers, for example.

The program has been validated against experimental data from 3 1/2" and 5" gauge locomotives and a limited amount of data from full size express locomotives. It appears that the size of

equipment influences some of the input variables but not the method of calculation.

FLUID PROPERTIES

There are two substances to consider – the flue gases and water as liquid, wet steam, saturated steam and superheated steam. Since temperature and pressure change throughout the heating cycle, it is necessary to calculate the properties of both fluids at various points throughout the process.

I have used various curve fits to published standard data which are summarised in Appendix 15. These curve fits have limited validity, but are adequate over the range of values found in small power boilers. Where appropriate, the curve fits are explicit, so that temperature may be determined from enthalpy and pressure or enthalpy determined from temperature and pressure, for example.

FIRE

The furnace performance is defined with the following settable parameters:

- Fuel chemical and proximate analyses, including moisture and ash contents.
- Unburnt fuel lost to smokebox or chimney before combustion.
- Combustion Efficiency (actually an indicator of furnace performance rather than coal quality.), defined as actual heat generated divided by the available high calorific value of the fuel.
- Air to coal mass ratio.

The combustion process is then modelled as follows:

1. Total fuel flow determined from grate loading x grate area.
2. Less fuel percentage lost before combustion.
3. Air flow determined from total fuel flow x stoichiometric ratio.
4. The combustion efficiency figure is used to determine how much carbon is burned to produce carbon monoxide to achieve the specified efficiency value.
5. The masses of the products of combustion (Carbon dioxide, carbon monoxide, water from hydrogen combustion + water from trapped moisture, sulphure dioxide) are determined using the atomic numbers of the relevant elements and the appropriate chemical formulae.
6. The gravimetric and volumetric composition of the flue gas is determined from the above, plus the known composition of air. The volumetric composition is useful for correlating the combustion process against test results obtained using Orsat or similar apparatus, for example.
7. The heat output of the fire is determined from the fuel flow, net of fuel lost before combustion, less latent heat of evaporation of water content as determined in Step 4 above.

The above steps are based on the method outlined in Reference 13.9.

FIREBOX HEAT TRANSFER

Firebed

Heat liberation in the firebed is determined by a settable parameter indicating how much heat is liberated in the firebed, the remainder being assumed to be liberated by combustion above the fire but within the firebox volume. Observation of a typical fire suggests volatile fractions and possibly small particles are burnt above the firebed.

The firebed model follows the method of Hall (Ref. 13.6). An iterative approximation determines a common firebed and flue gas temperature where the following conditions are satisfied

- ❓ The heat radiated from the firebed top surface is calculated from the Stefan-Boltzmann method. The emissivity of the firebed is a settable parameter. Hall assumed that the firebox walls absorb this heat. However, if flames within the firebox have significant absorptivity and emissivity that assumption is not valid. This is discussed in more detail below.
- ❓ The heat contained within the flue gas is calculated from the assumed inlet temperature and a settable constant value of flue gas specific heat. Flue gas specific heat changes by around 15% over the temperature range under consideration, but this is ignored for simplicity and a constant average value used.
- ❓ The firebed and flue gas temperature must be such that the total heat (radiant and flue gas heat value) are equal to the fire heat generated in the firebed. There seems to be no logical reason why the flue gas and fire bed should assume an identical temperature but it seems a reasonable working guess, given the mixing of the gas in the firebed.

Heat Transfer within the Firebox

Radiation Heat Transfer

If we assume a simple model where there is no flame action, all radiation emitted by the fire must be transferred to the firebox wall, since the walls completely surround the fire. The flue gas temperature would drop only from convective heat transfer to the firebox wall. This method of calculation was adopted by Hall (Ref. 13.3), however a more complete model is needed.

Diatomic gases such as N_2 and O_2 and hence air are usually taken to be neither absorbers nor emitters of infra red radiation. The usual products of combustion (CO_2 and H_2O) are weak absorbers and emitters according to the work of Hottel (Ref 13.11), although for the lengths involved in model work the effects would be very small.

However, flames behave as a strongly radiating gas, with emissivity dependent on beam length and probably other factors as well (Ref. 13.5). A flame with significant absorbtion and emission values will absorb heat from the firebed and transmit the remainder to the firebox wall. In addition, such a flame will radiate directly to the firebox. In particular, there is significant shielding of radiant superheater elements from the firebed, but also extra radiant transfer to the element by radiation from the flame; this is particularly relevant when considering the prediction of superheater performance.

It would be possible to assess radiation from flames and diatomic gases separately. However, it is simpler to consider the radiation effect of the mixture hereafter referred to as flue gas.

The author has reviewed two papers to better understand flame emissivity. (Refs. 13.12 & 13.13). Ref 13.12 deals with emission from forest fire flames. Emissivity from and transmission through

flames flames is usually estimated from an “absorbtion coefficient” and a “beam length” (or flame thickness) where:

and

Where:

ϵ = Emissivity [Dimensionless]

τ = Transmissivity [Dimensionless]

l = Beam length or flame thickness [metres]

x = Absorbtion coefficient. [per metre]

Since emissivity and transmissivity are a function of linear size, there are fundamental differences in the fire to firebox heat transfer between model and full size practice. The longer beam lengths in full size rail practice mean that flame emissivity and absorbtion is near unity, whereas in a 5” gauge model flame emissivity is around 0.2.

Ref 13.12 includes a review of previous work and identifies absorbtion coefficients ranging from 0.1 to 0.8 m^{-1} . The experimental work reported derives absorbtion coefficients of 2.24 to 3 m^{-1} depending on position within the flame at which tests were conducted. The paper is principally concerned with developing an experimental technique for measuring emissivity of forest fire flames.

Ref. 13.13 is a much earlier work conducted on four different pulverised coals in an industrial scale furnace. Emissivity values between 0.8 and 0.4 were typically observed. Absorbtion coefficients ranging from 0.0092 to 0.04 per inch (Equates to 0.36 to 0.78 m^{-1}) are reported, depending on position along the flame, flame thickness and source of coal.

Both the above references and Ref. 13.5 acknowledge that flame emissivity is not readily calculable and depends on at least the following factors:

- [?] Type of fuel, chemical composition, moisture content, particle size.
- [?] Position within flame – probably as free carbon is converted to ash and CO_2 along the flame length. Emissivity tends to be higher at the base of a flame, which is significant for small scale models where flame lengths are comparatively short.
- [?] Thickness of flame. Ref. 13.13 reports data on variation of absorbtion coefficient with flame thickness and flame length. This shows that the above simple equation for emissivity based on flame thickness and a constant absorbtion coefficient is not valid across wide variations in flame length or thickness.
- [?] Measurement method used to determine emissivity.

Notwithstanding the limitations noted above, the computer model uses an equation of the form to describe flame emissivity. It is further assumed that the absorbtion coefficient “ x ” is constant.

As described above, radiant heat emitted by the fire is attenuated by the flame before reaching the firebox wall, but the flame also emits radiant heat directly to the firebox wall:

Where:

I = Radiation intensity [kW]

τ = Transmissibility [Dimensionless]

And incorporating the Stefan Boltzmann equation, we get:

Where:

σ = Stefan Boltzmann constant [56.7×10^{-12} kW/m²/K⁴]

T = Absolute Temperature [Degrees Kelvin]

A = Surface area [m²]

Φ = Angle between normal to surface and line joining surfaces [Degrees or Radian]

X = Shortest distance between surfaces [m]

The final term of the above equation in square brackets is known as the “view factor” and describes how well the emitting surface can “see” the receiving surface. The computational problem is that the view factor is different for each part of the fire in relation to each part of the firebox wall; even if we split the fire into say 9 elemental areas and each firebox side into 9 areas, we would need to conduct 162 separate calculations, even allowing for symmetry. In order to get a reasonable estimate of flue gas temperature change, we would need many more firebox side elements and the problem becomes too complex for a spreadsheet solution. The problem becomes even more complex, because the flame is losing heat to the firebox wall and absorbing heat from the fire (since τ is less than unity), thus changing the flame temperature as it travels through the firebox; in order to calculate heat transferred to the firebox walls, we must keep track of the flame temperature at each point within the firebox.

In order to reduce the problem to manageable size, I have used assumed the firebox to be cylindrical with a diameter of equivalent area to the grate. The height of the cylinder is selected to give the actual firebox wall area. In addition, the fire is assumed to be concentrated at the centre point of the cylinder base.

Figure 1 - Assumed geometry of firebox model

In this model, the vertical firebox wall is split into a number of cylindrical sections (25 equal sections in the model), at a height above the fire. As an approximation, each section has one value of Φ_1 and for the firebox wall, $\Phi_2 = (90 - \Phi_1)$. The firebox crown is also considered as another discrete section. For each section, the flame absorbs energy from the firebed over a beam length R . Similarly, the flame radiates energy to the firebox wall over the elemental area cylindrical section, outlined in dotted in Figure 1.

The calculation for heat transfer to the firebox crown is similarly simplified and $\Phi_1 = \Phi_2 = 0$. There is a correction applied to the firebox crown area to subtract any area in shadow from radiant superheater elements.

Since radiant energy is absorbed by the flue gas along a beam length R , for each elemental wall area considered, it is necessary to consider how this affect the temperature distribution within the

firebox. The problem is illustrated in Figure 2 which shows a cross section through the firebox model.

Figure 2 - Showing how radiant heating of flue gas is apportioned to the firebox volume.

It will be seen that as radiant heat passes through a volume bounded by R_n and R_{n+1} on route to an elemental area of firebox wall, a calculable volume of gas will be heated. It is assumed that heat is dissipated in proportion to the length between the origin and the elemental wall area. Therefore, the amount of heat dissipated within the boundaries R_n and R_{n+1} can be calculated.

The flue gas temperature can be calculated from an energy balance of energy emitted and energy absorbed each discrete volume bounded by R_n and R_{n+1} . In addition, an allowance is made for combustion taking place above the fire, the heat release being assumed uniform over the 25 elemental volumes of the firebox.

The above calculation routine is a gross simplification of a very complex situation. It could be more accurately modelled using finite element techniques, however there is large uncertainty about an appropriate value for flame absorption coefficient as noted above. Therefore, the extra complication involved would probably not be justified.

In defence of the technique, approximating the fire to a single point means that half the real fire would have a better view factor to the firebox wall, and half would be worse. Similarly, approximating the firebox to a cylinder means that the corners of a rectangular firebox would have a worse view factor, while mid sides would have a better view factor. The assumption of perfect gas mixing within a volume is obviously false, but the average temperature of gas delivered to the flues will be approximately correct, even though some flues may receive hotter or cooler gas.

A value for flame absorption coefficient has been derived from Busbridge's test results and is around 0.7 m^{-1} . It is encouraging to note that using this same value for an analysis of a full size firebox, evaporation values of the right order indicated by Ref. 13.4 are predicted.

Convection Heat Transfer

A forced convective calculation is used, exactly the same as that for tubes and flues See section 6. An equivalent circular firebox is assumed, a mean velocity calculated and this is used to calculate Reynolds number, Nusselt number etc. No artificial increase in Nusselt is used in this calculation, since there is no turning flow involved to delay formation of a boundary layer.

Hall (Ref. 13.3) adopted a free convective calculation in this area, but since the fluid column is rising with an appreciable forced velocity, I consider a forced convection calculation more appropriate.

Summary of Firebox Heat Transfer Calculation

The detail of the calculation routine is as follows:

- [?] Firebed temperature and initial flue gas temperature is as calculation described in 5.1.**
- [?] For each section of firebox wall, starting from the base, calculate: Height up firebox wall, mean angle ϕ to centre of fire, area of firebox wall under consideration and view factor between the centre of the fire and area of firebox wall.**

- ❓ Calculate gas emissivity over beam length calculated above.
- ❓ Calculate gross radiation heat transfer from fire to firebox wall section, calculate net heat transfer from fire to firebox wall section. The difference between the two quantities is the energy absorbed into the firebox gas within that beam.
- ❓ Calculate the heat transferred by flame radiation acting on the elemental firebox wall area. The emissivity value is based on a beam length equal to the firebox diameter.
- ❓ Calculate the radiated heat transferred to the volume of flue gas bounded by R_n, R_{n+1} and the firebox wall by summing heat absorbed from all rays passing through that volume, proportioned by the volume of gas radiated.
- ❓ Calculate the convection heat transfer acting on the firebox wall bounded by R_n, R_{n+1} as discussed in Sections 5.2.2 and 6.
- ❓ Calculate the change in gas energy between R_n and R_{n+1} by adding (heat generated by combustion above fire + heat absorbed from the fire by radiation – heat transferred by flame radiation to firebox wall – heat transferred by convection to the firebox wall)
- ❓ Calculate the gas temperature entering boundary R_{n+1} by dividing above result by (gas specific heat x gas mass flow rate). This assumes that there is perfect mixing of gas within the volume under consideration, which is unlikely given the modest rising velocity of gas.
- ❓ Repeat calculation for section between R_{n+1} and R_{n+2} using new value for gas temperature.
- ❓ For total heat transfer by direct radiation from fire to firebox wall and flame radiation to firebox wall, determine evaporation rate of water from firebox surfaces.

The simplified firebox model also shows how flame and fire heat transfer are different between full size and miniature engine as follows:

	FULL SIZE GWR CASTLE CLASS				1/16 SCALE MODEL BRITANIA			
FIREBED TO WATER RADIATION	253	Kilowatts	10	%	2.85	Kilowatts	56	%
FLAME TO WATER RADIATION	2082	Kilowatts	83	%	1.48	Kilowatts	29	%
GAS TO WATER CONVECTION	164	Kilowatts	7	%	0.76	Kilowatts	15	%

Radiant heat transfer direct from firebed to firebox wall becomes more important as scale reduces, and less heat is passed forward to the flues.

Radiant Heat Transfer to Radiant Superheaters

The method described in Section 5.2 is adapted to determine the heat transfer to radiant superheater sections as follows:

- [?] Firebed temperature and initial flue gas temperature are as calculation described in 5.1.
- [?] Radiant superheaters are assumed to be located just under the firebox crown and directly above the fire. The mean angle φ to centre of fire is assumed to be zero. The view factor can thus be calculated. The radiated area of superheater is taken to be superheater diameter $\times \pi/2$. The average value of $\text{Cos}\varphi_2$ around the half circumference is taken as 0.5 and hence the average view factor across the “visible” element can be calculated.
- [?] Calculate gas emissivity over beam length from fire to superheater.
- [?] Calculate gross radiation heat transfer from fire to superheater, calculate net heat transfer from fire to superheater. No further allowance is made for energy absorbed into the firebox gas within that beam. This is offset by the firebox calculation having allowed for energy absorbed to the firebox crown full area. This is an approximation, but allows the calculation to be explicit, which it would not otherwise be.
- [?] Flame radiation is calculated using an emissivity value based on firebox diameter.
- [?] The superheater surface temperature is calculated by an iterative process as described in Section 8.2.

FIRETUBES

The calculation splits the firetube length into 25 sequential elements, so that the temperature changes across any single element are relatively small. The temperature of the boiler water is assumed constant, which is true to a good approximation for a boiling liquid. No account is taken of temperature drop through the metal tube wall, which is very small compared to the drop across the flue gas film.

Radiation Heat Transfer within Tubes & Superheater Flues

There is no direct radiation from the firebed to consider within tubes and flues and the beam length (generally taken as the tube diameter or hydraulic diameter for superheater flues) is relatively short. Therefore, flame radiation is a very weak effect in miniature engines, and a minor effect in full size. However, since the computation had been developed to look at this effect, it is retained in the computer model.

The calculation proceeds in the flow direction as follows:

- [?] Boiler water temperature determined from steam tables as summarised in Section 15.2.
- [?] For flue gas temperature as determined in the previous section (Firebox exit temperature for the first section of tube), the flue gas properties are determined as summarised in Section 15.1. Properties calculated: Bulk density, wall viscosity, Prandtl number, thermal conductivity.
- [?] Calculate Reynolds number from above properties and mass flow of flue gas per flue, flue diameter
- [?] Calculate friction loss coefficient, assuming smooth tube surface. Determine pressure loss of flow gas.

- ❓ Calculate Nusselt number from Dittus – Boelter equation if Reynolds number >2500 . Use relevant laminar correlation if $Re < 2500$. For both flow regimes, the boundary layer is assumed to commence at the flue inlet and progressively grow along the flue length. For laminar flow, the Sieder and Tate correlation is used, while a different correlation from Ref. 13.10 is used for turbulent flow. See Appendix 14 for further details.
- ❓ Calculate the wall heat transfer coefficient from the Nusselt number, thermal conductivity and tube internal diameter.
- ❓ Calculate heat transferred in the element, assuming the relevant temperature is the inlet temperature to the element.
- ❓ Calculate the radiated heat transferred in the section using the Stefan Boltzman equation and a calculated emissivity for the flue gases, based on tube or flue hydraulic diameter.
- ❓ From the energy balance, calculate the flue gas temperature loss across the element and hence the average temperature in the element.
- ❓ Re-calculate the heat transfer and temperature drop across the element. In a rigorous approach, all the above calculation steps should be repeated over several iterations of the process. However, the increase in accuracy is small and it is sufficient to re-calculate only the final step, provided short multiple elements are used.
- ❓ Repeat the calculation for the next element, using the output temperature from the previous calculation.

EVAPORATION AND STEAM CONDITIONS TO SUPERHEATER

The evaporation from each element of the boiler (firebox and flue tubes) is calculated using a heat balance and steam properties as determined in Section 15.2. It is assumed that steam is not generated in the fully dry condition – a very unlikely possibility in model boilers; the degree of wetness is specified as a program input and determines the enthalpy of the steam leaving the boiler.

The numerical model includes a section to represent the regulator, delivering steam to the superheater at reduced pressure. The regulator is modelled as a constant enthalpy expansion to the regulated pressure (a program input), resulting in a change of wetness and/or temperature in the steam delivered to the superheater. Steam properties are modelled as described in Section 15.2.

SUPERHEATER

Development of the superheater calculation routine has required more effort than the rest of the numerical model put together, which reflects the complexity of dealing with 3 concentric fluid streams and the heat transfers between them.

The calculation is able to work with flue and radiant type superheaters, if a non radiant superheater is required, the length of radiant superheater is set to a very small value. The calculation is complex since steam flows in 2 directions (from smokebox and to smokebox) in the respective superheater elements, but fire gases flow in 1 direction only. Therefore, initial conditions for both fire gases and steam are not known at either end of the superheater flue. To get round this problem, an initial calculation is performed, which assumes that the temperature drop in the flue gases is identical to that in a smoke tube (already calculated). This will be approximately true, given that pressure losses and hence flowrate of flue gas is approximately the same and provided that “conventional” diameters of superheater elements and flues. This initial

calculation provides an estimate of steam temperature in the wet side superheater at the plane of the firebox tubeplate.

The calculation assumes that heat transfer within the flue tube is by convection only and within the firebox volume is by radiation only. The convection calculation is more complex than the firetube one described above, because the steam side heat transfer coefficient is of a similar order the flue gas side, so both must be accounted for.

The full calculation process proceeds as follows:

Approximate Calculation of Wet Steam Temperature at Firebox Tubeplate

The calculation assumes that the temperature distribution along the superheater flue is identical to a smoketube, which has already been determined as described above. The veracity of this assumption is checked later in the calculation. The calculation is split into 5 elements and commences at the smokebox end of the flue, where the steam properties are known. It proceeds as follows:

- ❓ Calculate heat transfer coefficient per m run of superheater per degree C for superheater tube on flue gas side. Use pre-determined mean flue gas temperature and method as described in Section 6.
- ❓ Calculate heat transfer coefficient per m run of superheater per degree C for superheater tube on steam side. Method is similar to that on flue gas side, but with different fluid properties. The input steam conditions are as determined as described in Section 7.
- ❓ Calculate combined heat transfer coefficient per m run of superheater per degree C.
- ❓ Calculate heat transferred in the element, assuming the relevant inlet steam conditions to the element.
- ❓ From the energy balance, calculate the steam temperature loss across the element and hence the average temperature in the element.
- ❓ Re-calculate the heat transfer and temperature change across the element; in this instance the temperature change is a rise, because the calculation is proceeding against the flow direction. In a rigorous approach, all the above calculation steps should be repeated over several iterations of the process. However, the increase in accuracy is small and it is sufficient to re-calculate only the final step, provided short multiple elements are used.
- ❓ Repeat the calculation for the next element, using the output steam temperature from the previous calculation and the appropriate flue gas temperature.

Performance of Radiant Superheater Section

Calculation of the radiant superheater section commences at the wet steam side at the firebox tubeplate, using steam conditions as determined in the above section. The radiant superheater is split into 5 wet sections, the spearhead and 5 dry sections. In calculation of laminar heat transfer coefficients, the boundary layer is assumed to grow from the wet superheater header (start of previous section), and to be destroyed and commence growth from the spearhead. These should be reasonably good approximations, particularly in the case of the spearhead where the sharp turn & turbulence will separate any established boundary layer.

The calculation assumes a flame temperature which is derived from the firebox calculation (Section Firebed). The calculation of radiant heat input is described in Section 5.2.4.

The calculation proceeds as follows:

- ❓ An initial estimate of superheater wall temperature is required. This is calculated such that convective heat flow from the superheater wall into the steam is equal to radiative heat flow from the fire and flames to the pipe wall. This estimate is obtained by using a Newton Raphson iteration.
- ❓ For each elemental length of superheater element, the Nusselt number and convective heat transfer coefficient is calculated using the estimated pipe wall temperature. Heat transfer across the element is calculated for both the radiative and convective sides, and a revised wall temperature calculated for use in calculating the next element.
- ❓ Revised steam conditions are calculated from the heat balance for use in the next element.
- ❓ Pressure loss is also calculated across each element.
- ❓ The calculation is repeated for each of the 10 elements and the spearhead using the new values of steam conditions, pressure and pipe wall temperature. No heat transfer is assumed in the spearhead, but a steam pressure loss occurs.

Performance of Superheater Flue Section

Reynolds number within the superheater flue, comprising an annular space between multiple pairs of superheater elements and the flue is dealt with using the mean hydraulic diameter approach. No other shape correction factors are applied to the heat transfer coefficients to account for the unusual cross section. The calculations make use of the same heat transfer calculation methods already discussed, except that there is transfer to the wet steam superheater, dry steam superheater and flue wall to consider simultaneously.

As before, the calculation is split into 25 length elements along the total length of the flue and commences at the firebox tubeplate working toward the smokebox. For the wet steam superheater, temperature changes are increases, because the calculation is proceeding against the steam flow direction.

The calculation proceeds as follows:

- ❓ Steam conditions at start as determined in Sections 8.1 & 8.2, flue gas temperature as determined in Section FIRETUBES.
- ❓ For relevant wet and dry steam conditions and flue gas temperature, the fluid properties are determined as summarised in Section 15.1. Properties calculated: Bulk density, wall viscosity, Prandtl number, thermal conductivity.
- ❓ Calculate Reynolds number from above properties and mass flow of fluid, and flow area or hydraulic diameter.
- ❓ Calculate friction loss coefficients, assuming smooth tube surface. Determine pressure loss of fluids.
- ❓ Calculate Nusselt number from Dittus – Boelter equation if Reynolds number > 2500. Use relevant laminar correlation if $Re < 2500$. For both flow regimes, the boundary layer is assumed to commence at the flue inlet and progressively grow along the flue length. For laminar flow, the Sieder and Tate correlation is used, while a different correlation from Ref. 13.10 is used for turbulent flow. See Appendix 14 for further details.

- ❓ Calculate the wall heat transfer coefficients from the Nusselt number, thermal conductivity and tube internal diameter for flue gas-tube surface, wet steam-tube surface and dry steam-tube surface. An initial estimate of superheater wall temperature is required. This is calculated such that convective heat flow from the superheater wall into the steam is equal to radiative & conductive heat flow from the firebox flames to the pipe wall. This estimate is obtained by using a Newton Raphson iteration. Heat transfer coefficient for flue gas-boiler water is determined by the same method described in Section 6.
- ❓ Calculate heat transferred in the element, assuming the relevant temperatures at the inlet to the element.
- ❓ From the energy balance, calculate the flue gas temperature loss across the element and steam property changes in the element. Calculate the average temperatures in the element.
- ❓ Re-calculate the heat transfer and temperature drop across the element. In a rigorous approach, all the above calculation steps should be repeated over several iterations of the process. However, the increase in accuracy is small and it is sufficient to re-calculate only the final step, provided short multiple elements with relatively small temperature changes are used.
- ❓ Repeat the calculation for the next element, using the output temperature from the previous calculation.
- ❓ Upon completion of the calculation, the predicted wet steam temperature at superheater inlet is compared with the actual value to check the magnitude of any error. Typically, this is less than 10 Deg. C.

RE-CALCULATION OF FIRETUBE AND SUPERHEATER PERFORMANCE

After the first pass of calculation described above, there are still significant errors in the wet steam temperature obtained by back calculation through the superheater and predicted from the boiler pressure and regulator performance. Further, the relative flue gas flows through superheater flues and firetubes can be further refined, which in turn affects the predicted evaporation rate.

Therefore a further iteration of calculations described in Sections FIRETUBE to SUPERHEATER is undertaken with revised values for:

- ❓ evaporation
- ❓ gas flows through superheaters and firetubes
- ❓ wet side superheater steam temperature at firebox tubeplate plane.

validating the computer model

A complex calculation technique such as that described above must be validated against test results. Two sets of miniature tests have been published (Refs. 13.1 & 13.2), those by Busbridge being the better tests, while those of Ewins reported by Evans were obtained using a convenient, but error prone, test method.

Busbridge's Test on 3.5" Gauge "Brittania" Boiler

Busbridge carried out a well controlled series of tests on a Britannia boiler to LBSC's design and published the results in Reference 13.2. The tests looked at the boiler only and were conducted at the University of Cape Town.

Figure 3 - Comparison of Busbridge's test results with predictions from program.

Figure 3 shows good agreement has been achieved to Busbridge's tests across a range of grate loadings. The calculated stoichiometric ratio has been derived from Busbridge's measurements of carbon dioxide in the flue gas. It can be seen that the general pattern is for this ratio to asymptotically approach a value of around 15 at high grate loadings. At low grate loadings the ratio climbs to values near 30 or perhaps more.

The quality of prediction becomes worse at lower grate loadings, partly because variables in the program have been chosen to give better agreement at high grate loadings. However, the results suggest that free convection, probably in firebox spaces, become more significant at lower grate loadings. Since the program is intended as a design tool, accurate predictions at maximum grate loading are more important.

The computer confirms that flow in the fire tubes is indeed streamline (also known as laminar flow). Busbridge's tests included careful measurements on the non-radiant superheater performance, which is fortunate as this gives an accurate indication of heat transfer within the flues. The validation process also confirms D.E. Lawrence's opinion that the flow is not fully settled laminar flow, leading to much better convective heat transfer than well accepted correlations would predict. The results suggest that normal correlations of convective heat transfer need uplifting by about 60%. Possible reasons for this are:

- ❓ Gas flow from the firebox to the tubes is turning through 90 degrees, causing major disturbance to the boundary layers in the flues and hence improving heat transfer.
- ❓ Gas flow is well into the laminar regime (Reynolds number 500 to 1000 in a miniature firetube) but within the lengths of a miniature boiler, flow is unlikely to be "settled", giving thinner boundary layers and hence improved heat transfer.
- ❓ I don't accept Lawrence's opinion that "deposits" would cause such an increase in heat transfer, as Busbridge's tests were on a new boiler.

Ewins' Tests on 5" Gauge "Mona" Locomotive

Correlations against Ewins' tests are somewhat frustrating. The tests are subject to significant experimental errors and inconsistencies as follows:

- ❓ Flue gas volume was measured by catching the gas in a large plastic bag and then measuring the volume by a "water displacement method" – details not given. This seems very prone to large errors.
- ❓ The flue gas analyses are not consistent with a predominantly carbon fuel burnt in natural atmosphere. The calculations relating to flue gas analysis can be found at Ref. 13.9. These analyses are used to infer coal consumption and grate loading. This results in experimental error in interpreting how hard the engine was working. If the CO₂ and CO results are assumed correct when burning a good quality coal, the O₂ contents would be as follows:

TEST NUMBER	EXPERIMENTAL O ₂ BY VOLUME %	CALCULATED O ₂ BY VOLUME %
1	9	10
2	3.5	6.2
3	10	10.9

- ❓ The grate loading can be inferred from Ewins' values for "coal consumption per DBHP hour" and "DBHP per sq. ft of grate". If this is done, the predicted flue gas production do not match his measurement of flue gas production. Grate loadings some 12 to 18% higher than those stated would be required to generate the stated gas volumes. Only some 4 % could be accounted for by ash content and the calculation was conducted assuming no fuel is lost before burning, which is an implicit assumption in Ewins' data analysis.
- ❓ The smokebox and firebox draught measurements are not consistent with the inferred changes in grate loading. I am indebted to Duncan Webster for pointing this out.
- ❓ The gas temperature measurements in the firebox will be significantly in error due to the radiant heating from the fire. BS 2790 Appendix C states this error can be up to 300 Deg. C. Taking accurate temperature measurements in a highly radiant environment needs very specialised (and expensive) equipment.
- ❓ Multiple IMLEC results suggest a maximum efficiency of around 2.5 % is feasible on miniatures, while Ewins claims an efficiency of 4.9 % in Test 3, which illustrates the probable magnitude of experimental errors in Ewins' work.
- ❓ Some sources indicate the engine tested had thermal siphons in the firebox, but Evans in Ref. 13.1 , makes no mention of these. I have sought the original SMEE papers of Ewins' tests, but without success. If anyone is able to help with copies of these, I would be very grateful.

Nevertheless, I have attempted to compare some of Ewins' results with the numerical model. Grate loadings are based on assuming Ewins' readings of flue gas volume flow, CO₂ and CO readings are correct, but the grate loadings inferred from stated values of "coal consumption per DBHP hour" and "DBHP per sq. ft of grate" are also included. The following graph summarises the coal / air ratio, superheater and exhaust gas temperature results:

Figure 4 - Comparison Ewins' test results with predictions from program.

Figure 4 shows that Tests 2 & 3 were taken at very similar grate loadings, although the power produced by the engine varied over a 400 % range, confirming that the 5% claimed efficiency of Test 3, is not true and the boiler was not keeping up with steam demand. The air:coal ratio shows a drop from around 20 at low grate loading to 15 at moderate loading, which is consistent with Busbridge's result, but then a sharp increase; This suggest the fire may have had a hole in it, artificially increasing the air ratio.

The superheated steam temperature observed is some 45 to 50 deg. C higher than predicted, while the exhaust gas temperature is some 80 deg. C lower than predicted at the higher grate loading. The actual rate of evaporation cannot be inferred from Ewins' results.

I have also compared the predicted temperature profile in the firetube for Ewins' test No. 2, which appears to be the most reliable set of data. The results are shown in Figure 5, which shows the program predicts temperatures some 175 Deg. C higher than observed at the tube entrance, reducing to 45 Deg. C higher at the tube exit. Firetube exit temperatures observed by Busbridge were also considerably higher than those noted by Ewins.

Ewins' observed fire temperatures are far higher than my predictions (1600 Deg. C against 1150 Deg. C). If the fire were at the temperature Ewins claims the fire would be well beyond white heat. My calculation estimates the fire at yellow to white heat, which seems more logical. The experimental errors associated with measuring temperatures in highly radiant environments have already been noted. Therefore, it seems to me that Ewins' temperature readings must be treated with some suspicion, and consequently so must many of his other conclusions.

Figure 5 - Comparison of predicted and observed firetube temperature profile from Ewins.

Full Size Tests

In Ref. 13.4 I found a design chart for the temperature profile in boiler tubes. The chart, originally published in ALCO bulletin 2017, is based on a grate loading of 100 to 115 lbs./sq.ft./hour. Unfortunately, it does not state the relative areas of grate and tube gas flow area. However, I have tried reproducing the chart and have used boiler design details for the GWR Castle class, since I have that data to hand. This is probably not the best choice, since an American test was probably on a wide firebox boiler and certainly not burning best Welsh coal. The results of the comparison are shown in Figure 6, and considering the potential problems as noted above gives remarkably good agreement. Note particularly that the temperature leaving the firebox is very well predicted.

The flow in boiler tubes for full size loco boilers is turbulent, having a Reynolds numbers in the range 7500 at tube entry to 13000 at tube exit. By comparison, the equivalent numbers in a miniature boiler are typically 500 to 900 which indicate laminar flow. This means there are completely different flow patterns in full size and miniature work, and consequently the heat transfer rates are fundamentally different, which makes any attempt to draw parallels between full size and miniature dangerous; as LBSC said "you can't scale nature"!

To accommodate the difference in flow pattern and heat transfer, the program uses different formulae for heat transfer depending on the Reynolds number, so the comparison in Figure 8 exercises a different part of the program to that used for model comparisons. One of the tasks on my "to do list" is to obtain some of the Rugby and Swindon test plant data now held at York Railway Museum and use this to check and improve that area of the program.

Figure 6 - Comparison of ALCO Bulletin 2017 design values and calculated temperatures

Ref. 13.4 also includes a design figure for evaporation due to firebox walls, which is quoted at 55 lbs evaporation per square foot. The program predicts evaporation of 64 lbs/ sq.ft.; It may be that the design is a conservative value to give some safety margin in design, and there is also the issue of whether my assumptions match the original test boiler.

If anybody can help with a copy of ALCO Bulletin No. 2017, the author would be very pleased to hear from them.

THE "CONSTANTS"

There are several "constants" that the program requires to calculate boiler performance.

Grate Loading

Without some reasonable estimate of a realistic grate loading no calculations of heat production, gas flow, flow regime or heat transfer can commence.

The power output from a boiler clearly depends on how much fuel is put into it. However, with coal firing there is a limit to how much coal can be fed into a given grate; in full size rail practice it was reckoned that 100 to 120 lbs/sq.ft/hr could be fed before clinkering was likely to take place on express engines, approximately 50 lbs/sq.ft/hr was a more usual figure for freight work or shunting. There are no equivalent values for miniature practice, where one might expect a much thinner fire, and hence lower grate loading. Busbridge (Ref. 13.2) estimated that his tests could have been continued to higher grate loadings.

I tabulated results of various locomotive efficiency trials from pre 1967 to 2007 on engines from 3.5" to 7 1/4" gauge. Some 360 individual runs have been tabulated. I also collated as many design details as possible for some 30 locomotives across the same range of gauges. I was then able to calculate grate loading data for 114 runs on various designs.

I tried analysing this data set of 114 runs in various ways. Initially, I looked at a cumulative frequency graph which showed a reasonable Gaussian distribution with a median result of 33.4 lbs./sq.ft/hr., and an upper quartile value of 45.5 lbs./sq.ft/hr. However, the data is heavily skewed toward 5" gauge locomotives, mostly of main line outline.

One might expect a rising trend in grate loading with size, which is true as Figure 7 shows. There is, of course an extra point not shown on the graph at 56.5" gauge and 100 – 120 lbs/sq.ft/hr, which indicates that a curve of grate loading against gauge must flatten off from the trend shown in Figure 7. Clearly, we do not work our miniature engines harder than full size!

Figure 7 - Grate Loading Vs. gauge of engine

Some additional data can be found in Ref. 13.7, in which a "Rob Roy" was worked at 42.2 lb/sq.ft/hour. The author reported that 3 adults were being hauled and estimated the fire was being worked too hard. 3.75 lb of water was evaporated in this test. J. Busbridge (Ref. 13.2) estimated that during his tests on a 3.5" gauge Britannia boiler, the peak recorded firing rate of 20 lb/sq.ft/hour could have been easily exceeded.

For road use, a test on a full size Sentinel S6 steam waggon was reported in The Commercial Motor on 6th Jan. 1950. This used the usual Sentinel test route over Horseshoe Pass. 62.5 miles were covered in 3hrs. 2 mins. Using 4cwt. 13lb of coal on a grate of 3.28 sq. ft, hauling a gross load of 23tons 12cwt. This gives an average grate loading of 46.3 lbs./sq.ft/hr; as outlined above, one might expect a higher loading on a thicker full size fire, and a considerably higher value at peak loading.

I also tried a more sophisticated analysis of grate loading against Draw Bar Horse Power

developed per unit area of grate (a measure of how hard the locomotive is working compared to its grate size). However, the data showed very poor correlation, which was probably masked by the important variable "driver skill". This is not surprising given a 20:1 spread in measured locomotive efficiency over the various trials.

Several factors might affect the value for grate loading deduced from the above analysis:

- ❓ The data is based on a competition where the object is to use as little coal as possible.
- ❓ According to the competition rules, coal used during "waiting time" before the competitive run is debited against the competitors allowance.
- ❓ Coal may be lost "overboard" during the excitement of competition.
- ❓ The power output and hence boiler demand on a miniature locomotive is limited by the adhesive weight of the miniature, which in turn would limit the required grate loading. This would not apply to a road vehicle, where wheel slip is virtually unknown.

Based on all the above considerations, I estimate that a design loading of 40 lbs./sq.ft/hr would be reasonable for 5" and 7 1/4" gauge, and perhaps 20 to 25 lbs./sq.ft/hr as a more conservative value for smaller miniatures. There is not sufficient data to conclude much about larger boilers, except that the grate loading can be higher than 40 lbs./sq.ft/hr. For my own project of a boiler with a grate of just over 1 square foot, a value of 50 lbs./sq.ft/hr looks reasonable. So, grate loadings in miniatures are not proportional to Scale³, mainly due to overscale fire thickness, but the grate loading per unit area is significantly less than full size.

It may be that grate loading should really be related to cubic volume of firebed, so that lbs/hour/**cubic foot** of firebed would be quasi constant. In full size practice, I have seen fire thickness typically from 4" to 8". So based on full size values 100 lbs./sq.ft/hr would be equivalent to about 200 lbs./cubic ft/hr. In miniatures, we have a much thinner fire – a maximum of 1 1/2" – 2" in a 3.5" gauge Britannia, for example. So based on 200 lbs/**cubic ft**/hr and 1 1/1" to 2" fire thickness, grate loading would be 25 - 33 lbs./sq.ft/hr on a 3.5" gauge Britannia and around 50 lbs./sq.ft/hr on a deep firebox 5" gauge miniature. Figure 7 shows these estimates to be reasonable but somewhat higher than observed figures, which might be expected as they are based on the thickest possible fire.

Coal Lost Before Combustion

A remarkably large amount of coal is lost before combustion. This is probably due to the fierce draught through the fire carrying small coal particles away. The upward velocity through a full size fire is some 7 m/s (15 m.p.h.) which is quite a stiff wind, and able to carry small coal particles away. The velocity through a miniature firebed is around 1/3 of the full size value.

I have been able to infer values for fuel loss from Busbridges tests, Ell's tests on a GWR King Ref. 13.14 and a Professor Nicholson published a formula for predicting fuel loss in full size (Ref. 13.4). All of these results and prediction methods are shown in Figure 6, which shows:

- ❓ Professor Nicholson's method gives a proportional rise in percentage fuel loss with grate loading. However, Nicholson's constants predict a much higher rate of fuel loss than Ell measured.
- ❓ I have fitted a Nicholson type law to Ell's data, which is shown as "Full Size MJ Model", which roughly approximates to the measured data.
- ❓ The fuel loss for a miniature is much higher in proportion to grate loading than for full size, as shown by the much steeper line through fuel losses inferred from Busbridge's

data.

I have also tried plotting fuel loss against air flow through the grate, reasoning that it is air flow that carries the fuel away. However, it seems there are other factors affecting fuel loss and I am investigating effects of coal particle size and retention time within the boiler. In the interim, the following relationship which I have used to correlate Busbridge's data is an approximation to performance in miniatures:

Where:

Coal Lost = Coal lost before combustion as percentage of total coal fired. [%]

Grate Loading = Coal fired per unit area per unit time [kg/m²/s]

Figure 8: Percentage of fuel lost before combustion against grate loading.

Air Ratio

The amount of air reaching the fire governs combustion temperature, quantity of flue gas passing through the boiler and the quantity of heat ejected with the flue gas and is therefore an important variable in boiler performance. The quantity of air drawn in can be directly deduced from analyses of flue gas as made by Busbridge, Ewins and many workers in full size. Figure 7 shows miniature data from Busbridge and Ewins along with full size data from Ref. 13.14. The grate loading on the x axis is based on coal burned, not total coal fired.

Figure 9: Air flow through coal fired grates

The full size data shows a linear relationship between air demand and grate loading. The gradient of the relationship is about 90% of the stoichiometric air demand, but there is a significant air demand at zero grate loading. I have shown the line of stoichiometric air flow, based on a good quality anthracite, which is a rather steeper line than that calculated by Ell; I do not know the reason for the discrepancy. This zero intercept means that the air ratio (mass of air / mass of coal) tends asymptotically toward the stoichiometric value at high grate loadings. So changing the air flow must imply a change in combustion rate, so fiddling with dampers is not an option – at least in terms of steady state operation.

The miniature data shows similar trends, with the gradient of Busbridges air flow data being about 93% of the stoichiometric value but the zero grate load air demand is only a quarter of the full size value. This means that the miniature air to coal ratio tends toward a stoichiometric air ratio at much lower grate loadings. Ewins' results show much scatter but fall in a similar area.

Once again, there are clearly other effects that distinguish full size and miniature work; particle size, fire depth, grate geometry, edge effects around the grate are all possible influences. I intend

to undertake further theoretical investigations to see if a universal law for miniatures and full size can be developed. In the meantime, the air flow through a miniature grate can be calculated from:

Where:

Air Flow = Air flow through grate [kg/m²/s]

Stoichiometric Air Ratio = Theoretical mass of air to burn unit mass of coal

Coal Burned on Grate = Total coal fired less coal not burnt [kg/m²/s]

Areas are the grate area.

Other “Constants”

There are several other constants in the program that need some explanation or justification:

Fuel burnt above the grate – Not all the fuel is burnt on the grate, volatiles and light coal particles will burn in the firebox volume. In the analyses I have allowed for 10% of the burned fuel to be burned above the grate within the firebox. Combustion is assumed to cease in the tubes or flues.

Combustion Efficiency - From analysis of published tests, typical combustion efficiencies of 97.5% are appropriate to miniatures, giving carbon dioxide values between 0.5 and 1 %.

Absorption Coefficient of Flue Gas – This is a measure of the thickness of the “fog” of combustion gases and determines the ability of infra red radiation to pass through or be absorbed by the combustion gas. Work on combustion gases from forest fires and house fires has given absorption coefficients of around 0.8. I have found that 0.9 seems to work well for both miniatures and full size practice; the higher value is probably due to greater solids content in the flame of a forced draught coal fire.

Dryness fraction of steam – Analysis of Busbridge's superheater test results show that the steam must have been dry before entering the superheater. It may well be that a boiler can generate virtually dry steam, but Busbridge's boiler was set up on a bench. I would expect significant water carry over from a boiler travelling over imperfect track. For consistency, I have used a dryness fraction leaving the boiler of 99.9%. The program is then calculating a maximum estimate of superheat temperature.

INTERIM CONCLUSIONS

A workable prediction routine for model (and full size) firetube boilers has been developed. This calculates temperature and heat transfer profiles throughout the fire side surfaces of the boiler.

A design value of specific fuel flow rate of around 40 lbs/sq.ft/hour is appropriate to model boilers in the 5” to 7 ¼” gauge range. Full size locomotives used values of around 100 for express passenger work, while goods locomotives might be as low as 40 – 50.

In model work firetube diameter is governed principally by practical considerations of blockage. From a thermodynamic viewpoint, many small diameter tubes would always provide a better solution – giving the maximum area for heat transfer.

Due to the low heat transfer associated with laminar flow in a model firetube, a much larger l/d value than full size practice would be required to achieve the same percentage of transfer of total heat.

In full size work, the avoidance of laminar flow might provide a limit to the minimum firetube diameter, unless practical considerations impose a larger diameter limit.

For typical firetube geometries, the inlet and outlet pressure losses are dominant, hence the total vacuum required to induce flue gas flow across the firetube bank is relatively insensitive to tube diameter and length. It is however, very sensitive to the ratio gas flow area/grate area.

There is considerable scope for optimising the balance between superheating surface and evaporative surface within a given boiler volume. This is particularly so for models because:

- ❓ Laminar flow within superheater flues restricts the heat transfer.
- ❓ Small steam volumes and relatively large surface areas and low piston speeds mean cylinder condensation is an important issue leading to large values of the "lost quantity".

The computer model which has been developed tends to underestimate the heat transfer within a boiler.

The computer model uses a combined convective and radiant heat transfer calculation for all surfaces. There is a significant under prediction of heat transfer within flues of miniature boilers which is corrected by an uplift factor in the laminar flow regime. There are clearly other factors at work here which requires further work to investigate.

The program currently includes a very simple assessment of "missing quantity" based on correlation of the extensive experimental work of Bill Hall (Ref. 13.8). However, this is only applicable to miniature engines in the 5" gauge range. I hope to extend the calculation routine to determine the "missing quantity" for all sizes of engine.

Further work is also required to assess the energy lost in pumping flue gases (the blast pipe/ chimney combination), with a view to optimising the flue gas flow path.

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APPENDIX - RELEVANT HEAT TRANSFER EQUATIONS used in the computer model

Radiation Heat Transfer

Radiation heat transfer between two small black body surfaces (fire and firebox wall) can be calculated as follows:

Where:

Q = Heat flow [kW]

σ = Stefan Boltzmann constant 56.7×10^{-12} [kw/m² K⁴]

A = Surface area [m²]

T = Absolute temperature of surface [Deg. Kelvin]

Φ = Angle between normal to surface and line joining surfaces [Degrees or Radian]

X = Shortest distance between surfaces [m]

Suffix 1 relates to emitting surface, Suffix 2 relates to receiving surface.

The term is known as the "view factor" and describes how well the emitting surface can see the receiving surface.

A numerical integration must be conducted across the two surfaces A_1 and A_2 , such that changes in Φ across each element are small. This can become very complex for geometries such as a fire surface and firebox.

Forced Convective Heat Transfer to or from a Tube

Where:

Re = Reynolds number

ρ = Fluid density [kg/m^3]

V = Mean fluid velocity [m/s]

D = Pipe diameter [m]

η_b = Bulk Fluid viscosity [$\text{N m}^2/\text{s}$]

[Watts/ m^2/K]

Where:

Nu_D = Nusselt number, determined as below [Dimensionless]

k = Thermal conductivity (Fluid property = function of temperature and pressure) [Watts/m/K]

[Watts]

Where:

T1 = Temperature of Wall [Degrees K]

T2 = Temperature of Fluid [Degrees K]

L = Length of pipe [m]

Laminar Flow

Laminar flow is assumed to take place when Reynolds number is less than 2500.

The Sieder & Tate Correlation is used which contains the term to account for the growth of the boundary layer and implicitly assumes that the boundary layer grows from the start of the section under consideration. This is not true when pipes are considered as a number of discrete axial elements. Therefore a modification of the Sieder Tate correlation must be used to apply between points L_1 and L_2 from start of the boundary layer:

If the above formula gives a value less than 3.66, then Nu_D is set to 3.66.

Where:

Pr = Prandtl Number for fluid [Dimensionless]

η_b = Bulk viscosity at mean fluid temperature [$N\ m^2/s$]

η_w = Viscosity at wall temperature [$N\ m^2/s$]

L_1 = Length from commencement of boundary layer at start of section [m]

L_2 = Length from commencement of boundary layer at end of section [m]

It will be seen that the formula reduces to the Sieder Tate formula when $L_1 = 0$

Turbulent Flow

Turbulent flow is assumed to take place if $Re > 2500$.

The Dittus Boelter equation is used:

Where:

n = 0.4 when pipe wall is hotter than fluid [Dimensionless]

= 0.3 when pipe wall is cooler than fluid [Dimensionless]

Corrections for entry length and boundary layer growth are applied to the above as follows taken from Ref. 13.10:

Where:

D_i = Internal diameter of pipe [m]

L = Length from start of boundary layer growth [m]

The Dittus Boelter equation is normally considered applicable for $Re > 10,000$, but is used for models at Re of around 8000. Comparison with other more complex formulae show little advantage over the Re range down to 2500, so the simpler formula is used.

Pressure Losses

Pressure Losses are calculated as follows:

Where:

ρ = Fluid density [kg/m³]

V = Fluid velocity [m/s]

APPENDIX – FORMULAE ADOPTED FOR CALCULATING FLUID PROPERTIES

Flue Gases

The pressure throughout the combustion circuit departs only marginally from atmospheric. According Rogers and Mayhew "Thermodynamic and Transport Properties of Fluids", data for dry air at low pressure can be used with reasonable accuracy for carbon monoxide and nitrogen as well. Since air is 80% nitrogen and air / fuel ratios rarely approach stoichiometry, the flue gas must also be mostly nitrogen and therefore, data for dry air has been used throughout. The following formulae have been fitted to the data for transport properties:

Specific heat C_p = Assumed constant 1.11 [kJ/kg K], this value is variable in the program.

Dynamic Viscosity $\mu = -8.8144E-12 \times T^2 + 3.8157E-08 \times T + 1.8526E-05$ [Ns/m²]

Thermal Conductivity $k = 5.0069E-05 \times T + 3.0201E-02$ [w/m K]

Prandtl Number $Pr = -1.2044E-10 \times T^3 + 2.4927E-07 \times T^2 - 1.0926E-04 \times T + 6.9512E-01$ [Dimensionless]

Density $\rho = 20.583 \times T^{-0.619}$ [kg/m³]

where T is temperature Degrees Celsius.

Steam Properties

Water

Specific heat C_p = Assumed constant 4.19 [kJ/kg K] Assumed constant.

Heat energy $U_f = C_p \times T$

Specific Volume $v = 0.001$ [m³/kg] Assumed constant.

Specific Enthalpy $h_f = U_f + P \times v_f / 1000$

where T is temperature [Degrees Celsius]

Saturated Steam

Saturation Temperature $t_s = 98.6 \times P^{0.2588}$ [Degrees Celsius]

Specific Volume $v_g = 1.7023 \times P^{-0.9421}$ [m³/kg]

Specific Enthalpy $h_g = 2677.2 \times P^{0.0161}$ [kJ/kg]

Dynamic Viscosity – See under “Superheated steam”

where P is absolute pressure [Bar]

Wet Steam

For steam of Y dryness:

Specific Internal Energy $U_W = Y \times U_g + (1-Y) \times U_f$ [kJ/kg] where U_g & U_f are for the saturated condition at relevant pressure & temperature.

Specific Volume $v_W = Y \times v_g + (1-Y) \times 0.001$ [m³/kg] where v_g is for the saturated condition at relevant pressure.

Enthalpy $h_W = Y \times h_g + (1-Y) \times h_f$ [kJ/kg] where h_g & h_f are for the saturated condition at relevant pressure & temperature.

Dynamic Viscosity – Taken as for vapour content, see under “Superheated steam” for calculation method.

Superheated Steam

For steam at temperature T [Degrees Celsius] & absolute pressure P [Bar] the following formulae give reasonably accurate predictions:

Specific Internal Energy $U_g = -7.0755 \times P + 2360.1 + (0.0157 \times P + 1.5172) \times T$

Maximum error = 0.3% over the range 1 to 15 Bar & 150 to 400 Deg. C.

Specific Volume $v_g = R' \times (T+273)/P$

Where $R' = 0.004625 - (1.4512E+06 \times (T+273)^{-4.0097}) \times P$

Maximum error = 0.4% over the range 1 to 15 Bar & 150 to 400 Deg. C.

Enthalpy = $H_g = -9.0877 \times P + 2486.6 + (0.0199 \times P + 1.9776) \times T$

Entropy = $S_g = (0.0042 \times T + 6.9972) \times (P^{-0.06532})$

Maximum error = 1.9 % over the range 1 to 15 Bar & 150 to 400 Deg. C and within 0.5 % over most of the range.

Prandtl $Pr = (5.5085E-07 \times P - 1.5237E-09) \times T^2 + (-3.8915E-04 \times P - 9.9678E-05) \times T + (6.8918E-02 \times P + 9.6415E-01)$

The above formula gives values to within 4% over the full range saturation temperature to 400 Deg. C and 1 to 15 Bar Abs.

Thermal Conductivity $k = (6.0990E-09 \times P + 5.8342E-08) \times T^2 + (-5.4470E-06 \times P + 7.2538E-05) \times T + (1.278E-03 \times P + 1.614E-02)$ [w/ m K]

Dynamic Viscosity $\mu = 2.5781E-13 \times T^2 + 4.1392E-08 \times T + 7.8303E-06$ [Ns/m²]

The above formula is a good fit to data at 4 Bar and is assumed independent of pressure, which is true within 4% over the full range saturation temperature to 400 Deg. C and 1 to 15 Bar Abs.